Roll No. ....

72601

M.Sc. Physics 1st Sem.

Examination–December, 2014

Mathematical Physics

Paper : I

Time : 3 hours ***Max. Marks : 80***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

Note : Attempt **five** questions in all, selecting **one** question from each Unit.

UNIT – I

1. (i) Prove that eigen value of a skew Hermitian matrix is either zero or purely imaginary. 4

(ii) Derive Rodrigue's relation for Legendre polynomials. 4

(iii) Explain convergence property for solutions of Legendre equation. 4

(iv) Using Fourier Series, show that : 4

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n} = \frac{x}{2}$$

UNIT – II

2. (i) Explain eigen values and eigen vectors of a matrix. Find the eigen values and a set of mutually orthogonal eigen vectors of 12

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(ii) Prove that the eigen vectors corresponding to distinct eigen values are linearly independent. 4

3. (i) Prove that any two eigen vectors corresponding to two distinct eigen values of a Hermitian matrix are orthogonal. 4

(ii) For the vector space R^4 of elements $[x_1, x_2, x_3, x_4]$, show that the vectors which satisfy the relation

$$2x_1 - 3x_2 - x_3 + x_4 = 0$$

$$x_1 + x_2 + 2x_3 - x_4 = 0$$

from a subspace of R^4. Find a basis for this subspace. 12

UNIT – III

4. While describing series solution around a point, obtain complete solution of :

$$x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + ay = 0 \text{ ; } a = \text{constant.}$$

How classification of a point is made ? Give one example of each type. 16

5. (i) Determine the nature of a point at infinity in respect of :

(a) $(1-x^2)\dfrac{d^2y}{dx^2} - \dfrac{xdy}{dx} + n^2y = 0;$

n = integer

(b) $x(x-1)\frac{d^2y}{dx^2} + [(1+a+b)x - C]$

$\frac{dy}{dx} + aby = 0$ 6

(ii) Prove that Bessel Equation can be derived from Legendre equation. 10

UNIT – IV

6. (i) Show that : 4

$$H_n(x+y) = 2^{-n/2} \sum_{p=0}^{n} \frac{n!}{p!(n-p)!} H_{n-p}(x\sqrt{2}) H_p(y\sqrt{2})$$

(ii) If $y = \exp(-x^2/2) H_n(x)$ and

$$I_{m,n} = \int_{-\infty}^{\infty} y_m(x)\, y_n(x)\, dx$$

then prove that $I_{n,n} = 2n I_{n-1,n-1}$; H_n is Hermete polynomial. 4

(iii) Explain integral representation of Bessel functions. Deduce value of 8

$$J_{\pm 1/2}(x) \wedge J_{\pm 5/2}(x)$$

7. (i) Derive the relation :

$$J_n(x) = (-2)^n x^n \left\{ \frac{d^n}{d(x^2)^n} \right\} J_o(x) \qquad 4$$

(ii) Prove that : 4

$$\int_0^{\pi/2} J_1\,(x \cos\theta)\, d\theta = \frac{1 - \cos x}{x}$$

(iii) Derive the Rodrigue's relation for Laguerre polynomials and verify the orthogonality of Leguerre functions. 8

UNIT – V

8. (i) Laplace transform of the displacement function y(t) for a forced, frictionless, spring–mass is found to be 8

$$y(s) = \frac{w\,F_o/M}{(s^2 + w_o^2)\,(s^2 + w^2)}$$

for a particular set of initial conditions. Find y(t).

(ii) If S_n and C_n denote finite Fourier sine and cosine transforms in the interval $0 \le x \le \pi$, prove that 8

(a) $S_n\left\{\frac{x}{n}\right\} = \frac{1}{n}\sqrt{\frac{\pi}{2}}\,(-1)^{n+1}$ n is an integer

(b) $C_n\left(e^{an}\right) = -\sqrt{\frac{2}{\pi}}\,\frac{a}{n^2 + a^2}\left[1 + (-1)^{n+1}\, e^{a\pi}\right]$

9. (i) Show that Fourier transform of a Hermite-Gauss function

$$U_n(x) = H_n(x)\, e^{-x^2/2}, \quad x = 0, 1, 2, 3 \ldots.$$

is a Hermite Gauss function within a constant. 10

(ii) Show that Laplace transform of Laguerre polynomial

$L_n(at)$ is given by

$$L\left[L_n(at)\right] = \frac{(s-a)^n}{s^{n+1}};\ s > 0$$ 6